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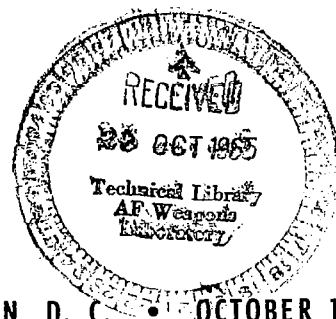
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## A MICROWAVE LENS SYSTEM FOR PLASMA DIAGNOSTICS

*by R. A. Bitzer and E. H. Holt*

Prepared under Grant No. NsG-48 by  
RENSSELAER POLYTECHNIC INSTITUTE  
Troy, N. Y.  
for



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## SUMMARY

This report discusses the use of lenses to provide plane waves at microwave frequencies for plasma probing. Based on the requirements dictated by an experimental objective of measuring the Faraday rotation of the polarization of an electromagnetic wave in a magnetoplasma, two types of lenses were designed. These were the dielectric lens and the Fresnel half-period zone plate. Tests were conducted on lenses built to these designs and the results obtained were close to the design figures. The superiority of the Fresnel plate over the dielectric lens for plasma diagnostics is demonstrated. The lenses were built for use at K-band frequencies (20 Gc) but improved diagnostics of a plasma can be expected if lenses built to these designs are used at higher frequencies.

## INTRODUCTION

A lens can be used to transform a non-planar wave front into a plane wave front by means of reflection, refraction, diffraction, or a combination of these. In the early days of radio the quasi-optical properties of radio waves were studied by such men as Lodge, Fleming, Righi, and Bose (ref. 1). Lodge and Fleming separately found difficulty in locating a focal point for their lenses. This was attributable to the fact that the dimensions of their lenses were comparable to the wavelength used. Such gain as was obtained was due to the lens acting as a diffracting obstacle. In Italy, Righi made a lens out of paraffin and sulphur. It was 32 cm in diameter, 7 cm thick at the center and had a focal length of 50 cm. Fairly good results were obtained at a wavelength of 3 cm.

As radio developed lower frequencies were found more reliable for long distance communication and the interest in the quasi-optical region disappeared. It was not until the use of radar in World War II that interest revived. Considerable work has now been done on microwave lenses (refs. 2,3,4) and extremely sharp resolutions have been obtained. These lenses are not only used for detection systems and communication purposes, but they have also been used with more exotic systems such as plasma resonance scanners (ref. 5) and interferometers. The lenses to be described here were designed for plasma diagnostics.

## A Lens System for Plasma Diagnostics

To be able to measure the properties of plasmas as the electron density of the plasma increases, there must be a corresponding increase in frequency of the microwave probe system. At X-band frequencies a special waveguide section can be used to house the plasma cell, whereas at K-band frequencies the dimensions of the waveguide are inconveniently small to house a plasma cell.

For this reason a lens system is one solution to the problem since it relaxes the restriction on the size of the plasma cell. For our particular application, it was envisioned to direct a beam of microwave energy through the plasma and to detect it on the other side by means of a microwave polarimeter. The polarimeter is used to measure the Faraday rotation of the wave as it propagates through the plasma in the presence of a magnetic field.

The lens system consists of a primary feed in the form of a conical horn and a secondary radiator which is the lens itself. Two types of lens were chosen for study. One was a solid dielectric lens with a hyperbolic contour on one side and a planar contour on the other side, and the other was a half-period zone plate. The zone plate is patterned after the optical Fresnel plate. It is not really a lens at all but is characterized as such because it has a similar focusing action.

The design data for the lenses were taken at K-band frequencies, but the principles of design are developed in general terms so that similar lenses can be designed for other frequencies of interest.

The general principles of microwave lens systems for plasma diagnostics are discussed by Heald and Wharton (ref. 6). A more complete description of the use of dielectric lenses is given in this reference than we are concerned with here where the work with the dielectric lens is for the purpose of making a direct comparison with the Fresnel plate systems.

#### THE CHOICE OF A SUITABLE MICROWAVE LENS SYSTEM

The general requirement of a lens system for magnetoplasma diagnostics is that it should not discriminate against any particular type of polarization of the wave. This means that the system must have axial symmetry. An additional requirement for the primary radiator is that it should have a fairly broad radiation pattern with little or no side lobe distribution. The conical horn with standard gain, that is, without a long taper, fulfills these requirements. The secondary radiator is required to convert the spherical wave front from the horn to a plane wave front for transmission through the plasma. The receiving lens is required to have the reverse characteristic.

Four types of microwave lenses may be distinguished. The metal plate lens consists of a set of conducting strips that are placed parallel to the electric field of the transmitted wave, and are spaced a little more than a half wavelength apart. The strips act as waveguides and as the phase velocity in a waveguide is greater than the phase velocity in free space the design can be such that the phase of the overall emerging wave is constant. E plane or H plane waves can be propagated. However the action of the lens is clearly dependent upon the polarization of the wave so it is not suitable for our present application. The artificial dielectric lens is made out of dielectric material with strips, spheres or disks of another material embedded in it. The size of these foreign objects should be small compared to the wavelength. Their effect is to reduce the weight of the lens compared with one made out of pure dielectric. However, the solid dielectric lens has the advantage that it does not suffer from the frequency sensitivity of the artificial dielectric lens. In addition its action does not depend upon the wave polarization, in

contrast to some forms of artificial dielectric lens. It was chosen as one type of lens to be tested.

The Fresnel half-period zone plate has the appearance of alternate opaque and transparent concentric rings. It operates on the principle that a series of diffractions of an incident wave produces constructive interference. If a point source is used at the focal point on one side of the plate a wave front will be produced on the other side that will exhibit constant phase. The designation half-period comes from the fact that the radii of the concentric rings are so placed that the phase of the incident wave will change every half-period. The Fresnel plate has the disadvantage that it is frequency sensitive. It is axially symmetric and was chosen to be tested.

## DIELECTRIC LENSES AND FRESNEL PLATES

Two requirements are imposed on our choice of the secondary radiator. First, the spherical wave front coming from the horn must be converted to a plane wave front and visa versa for the receiving end. Second, the lens chosen should not discriminate against any particular type of polarization of the wave, and must therefore have axial symmetry. It was found that only two types of lens out of the number available can meet both requirements; the dielectric lens and the Fresnel half-period zone plate.

A dielectric lens and three Fresnel plates of different diameters were built in order to compare the experimental results with the calculated values from design considerations. One Fresnel plate was constructed to the same dimensions (10.5 inch diameter) as the dielectric lens so that the properties of each could be compared. Two 22 inch Fresnel plates were built, one with alternate rings that were transparent and opaque to the microwave energy, the other made as a folded zone plate where the entire plate was transparent to the microwave energy. The purpose of the two larger plates was twofold; to be able to check the gain of the lenses by means of comparison instead of the reflection method, and to check the phase of the wave front by comparing radiation patterns taken at close and long range.

The microwave apparatus used in the experiments is shown in Figures 1 and 2. Radiation patterns, polarization measurements, gain measurements, and power distributions were taken. The latter two measurements were confined to the 22 inch

Fresnel plate because it is this plate that is designed to give optimum results and could be scaled down for use on 4mm. wavelengths. The scaled down version should give the same half-power beamwidth and radiation pattern as the 22 inch diameter plate.

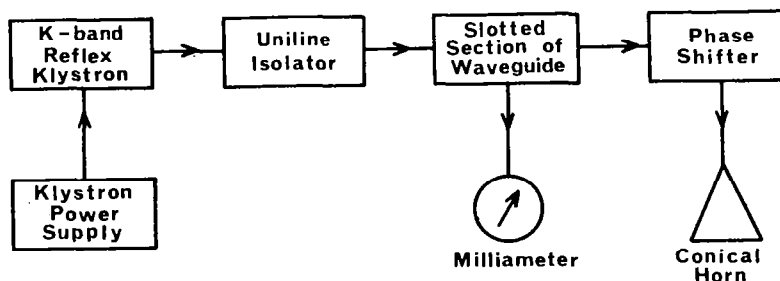


Figure 1 Transmitting Section for Measuring Lens Properties

The dielectric lens operates on the principle that a wave travelling through the lens has a velocity of propagation that is less than its velocity

in free space. The uniqueness of this lens is that the index of refraction does not change as the wave passes through. Rather it remains fairly constant at a value greater than one.

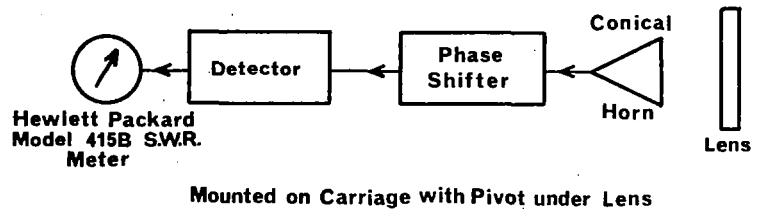


Figure 2 Receiving Section for Measuring Lens Properties

The lenses can have several types of contours depending on size, aperture illumination, aberration corrections, etc. The most common types are hyperbolic, aplanatic, and stepped hyperbolic lenses. A hyperbolic dielectric lens was constructed and tested for freedom of polarization. Its radiation pattern was also plotted.

### Dielectric Lens Design

The design of the lens is based on the principles of geometrical optics. Optical ray paths of pairs of wave fronts are to be kept equal as the rays pass from air, through the dielectric, and into air again. The rays are bent as they pass through the dielectric in which the refraction is governed by Snell's Law of Refraction. In order for the rays to emerge parallel from the plane side of the lens, the side of the lens towards the source is curved in the shape of a hyperboloid as illustrated in Fig. 3.

A simplified design procedure is followed here. More comprehensive discussions can be found in references 7 and 8. The following lens parameters are given and illustrated in Fig. 3.

Given:

1. Index of refraction,  $n$ .
2. Focal length,  $L$ .
3. Maximum angle between radius of curvature and axis of lens,  $\theta$ .

Find:

1. Different radii of curvature using different values of  $\theta$ .

$$r = \frac{(n-1) L}{n \cos \theta - 1}$$

2. Diameter of lens.

$$D = 2 r \sin \theta_{\max}$$

where

$$\theta_{\max} = \cos^{-1} \left[ \frac{(n-1) L + r_{\max}}{n r_{\max}} \right]$$

3. Thickness of the lens at its axis.

$$t = r \cos \theta - L$$

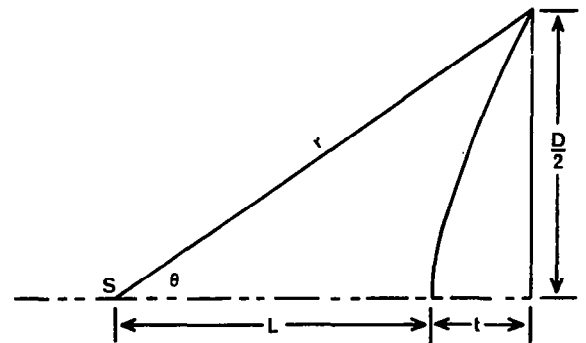


Figure 3 One Surface Lens with Hyperbolic Contour

Using these relations, a lens with the following dimensions and characteristics was constructed.

Focal length, L: 8 in.

Maximum radius,  $r_{\max}$ : 9.80 in.

Maximum thickness, t: 1.00 in.

Diameter, D: 7.40 in.

Maximum angle,  $\theta_{\max}$ :  $22^\circ$

Material: Methyl methacrylate (Plexiglass) with  $n = 1.674$ .

If the maximum angle  $\theta$  is  $20^\circ$ , then the field across the aperture is nearly uniform resulting in large side lobe distribution in the radiation pattern. If the maximum angle  $\theta$  is  $40^\circ$ , the field across the aperture becomes tapered, causing lower side lobe distribution.

The maximum angle  $\theta$  is about  $20^\circ$  on our lens, thus we expect a large side lobe distribution in the radiation pattern. Since the lens exhibits constant amplitude distribution across the aperture and constant phase distribution, the secondary radiation pattern of the aperture can be found from the equation of the far field radiation pattern.

The calculations to find the amplitude distribution of the lens can be obtained from the power distribution across the lens since these two quantities are directly related. The relative aperture power to field distribution is given by (refs. 3,4)

$$\frac{P(\rho)}{P(\theta)} = \frac{\sin \theta d\theta}{\rho d\rho}$$

where  $P(\rho)$  is the power per unit aperture radius at a distance from the axis, and  $P(\theta)$  is the power per unit solid angle radiated from the point source.

Since

$$r = \frac{(n-1) L}{n \cos \theta - 1}$$

and

$$\rho = r \sin \theta$$

then

$$\rho = \frac{(n-1) L \sin \theta}{n \cos \theta - 1}$$

$$d\rho = \frac{(n-1) L (n - \cos \theta) d\theta}{(n \cos \theta - 1)^3}$$

Substituting back into the power ratio relationship,

$$\frac{P(\rho)}{P(\theta)} = \frac{(n \cos \theta - 1)^3}{L^2 (n-1)^2 (n - \cos \theta)}$$

Solving the above relation for  $P(\rho)$  and substituting the values for  $n$  and  $L$  from the lens design,

$$P(\rho) = \frac{(1.674 \cos \theta - 1)^3 P(\theta)}{28.7 (1.674 - \cos \theta)}$$

Substituting in values of  $P(\rho)$  for various corresponding angles  $\theta$  from the H plane pattern of the conical horn, we see (Table 1) that the power is fairly uniform across the aperture. Since the amplitude distribution of the field is related to the square root of the power distribution, the amplitude distribution is also constant.

Angle in degrees	0°	5°	10°	15°	20°
Power per unit area across the aperture (db)	0.000	-0.0074	-0.0271	-0.0595	-0.0750

Table 1. Distribution of Power across the Aperture of the Dielectric Lens

Solving for the far field radiation pattern, points are plotted which can be superimposed on the experimental radiation pattern obtained as illustrated in Fig. 4. From the calculated values, the pattern has a fairly large side lobe distribution as postulated, but it appears that at certain points the experimental and calculated values disagree. This can be attributed to phase error, probably of the quadratic variety. One cause of the phase error is the proximity of the lens receiving system to the horn of the transmitting system, since the distance between them is the transition distance separating the Fresnel from Fraunhofer regions. Another cause of phase error is the "spillover" effect of electromagnetic energy over the edges of the lens. These errors can be largely reduced by (1) increasing the separation of the lens under test from the transmitting horn, and (2) by making the lens aperture large enough to capture most of the energy, or by making the horn more directional so that a smaller aperture can be used.

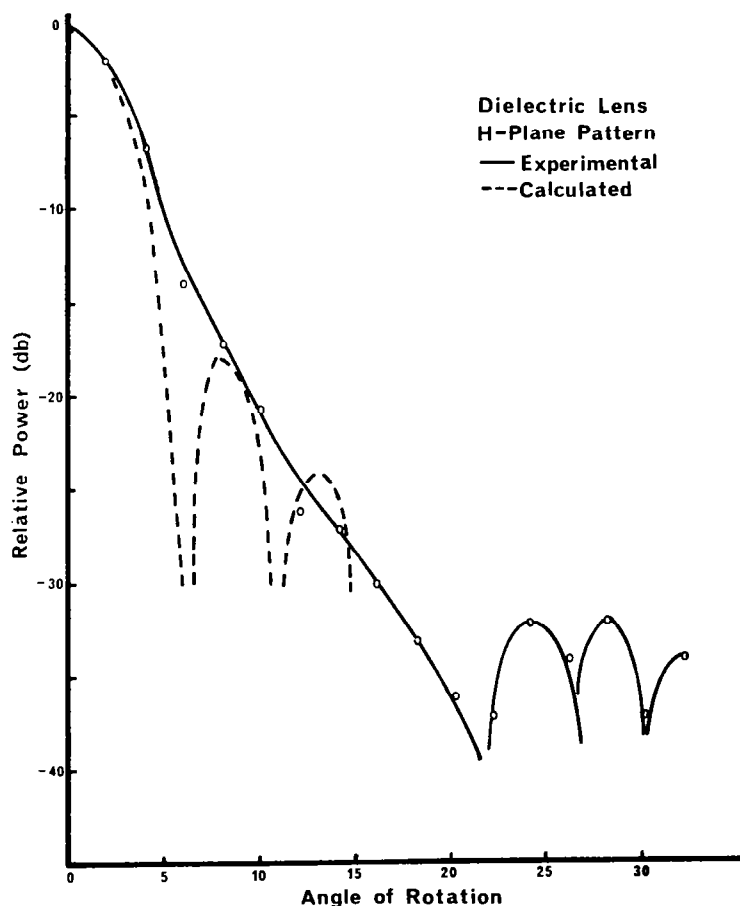


Figure 4 Radiation Pattern for Dielectric Lens.

During operation, phase shifters were inserted in both the receiving and transmitting ends. At 90° apart, circular polarization of the wave was evident as a maximum was indicated at the receiver. As the angle

varied from 90 degrees to 0 degrees apart it was confirmed that the lenses would accept any type of polarized wave.

### Design of the Fresnel Half-Period Zone Plate

The Fresnel plate is an electromagnetic diffraction device capable of focusing a plane wave at its focal point. The plate can be designed to have concentric rings which are alternately opaque and transparent to the electromagnetic waves. Such a plate was built using alternate rings of aluminum foil glued to a plexiglass backplate.

Another type of construction is referred to as the folded zone plate. This plate has the advantage of increased gain. Instead of reflecting the energy from the aluminum strips, the energy is made to reverse its phase by half a wavelength and, passing through the plexiglass, reinforces the other wave fronts on the other side of the plate. For the zone plate, the distance from the focal point to the edges of successive zones must be integral multiples of a half wavelength

$$\rho_N = L + \frac{N\lambda}{2}$$

where  $\rho_N$  is the distance from the focal point to the  $n^{\text{th}}$  zone edge.

$L$  is the focal length.

$\lambda$  is the wavelength of the electromagnetic wave.

The radius of the  $N^{\text{th}}$  edge is

$$r_N = \sqrt{N L \lambda + \frac{N^2 \lambda^2}{4}}$$

This relation holds for either the folded zone plate or the transparent and opaque plate. The focusing action is indifferent on the latter whether the center zone is transparent or opaque.

The gain for the alternate ring plate expressed in decibels can be expressed as follows (ref. 9):

Gain for transparent center zone plate

$$G = 20 \log \left\{ \frac{4 \frac{L}{\lambda} + 1}{4 \frac{L}{\lambda} + 1/2} \sum_{n=1}^N \frac{4 \frac{L}{\lambda} + 2n - 3/2}{2 \frac{L}{\lambda} + 2n - 3/2} \right\}$$

Gain for opaque center zone plate

$$G = 20 \log \left\{ \frac{4 \frac{L}{\lambda} + 1}{4 \frac{L}{\lambda} + 1/2} \sum_{n=1}^N \frac{4 \frac{L}{\lambda} + 2n - 1/2}{2 \frac{L}{\lambda} + 2n - 1/2} \right\}$$

The gain relationships for the folded zone plate are somewhat different in form. The gain for the plate is

$$G = \frac{4\pi R(N)}{\Omega_2}$$

where  $\Omega_2$  is the solid angle subtended by the zone plate as viewed from the focal point. If a plate has a focal length  $L$  and diameter  $D$ , then

$$\Omega_2 = 2\pi - \frac{4\pi L}{\sqrt{4L^2 + D^2}}$$

In terms of the wavelength and the number of half-period zones, the expression for the gain becomes

$$G = \frac{2(2L + N\lambda)}{N\lambda} R(N)$$

In these expressions for gain, there is a factor  $R(N)$  which is the ratio of the intensity at the focus of a zone plate to the intensity at the focus without the zone plate. For a certain design frequency  $f$ , the ratio is

$$R(N) = 4 \left\{ \sum_{n=1}^N \frac{\frac{8L}{\lambda} + (2n-1)}{\frac{8L}{\lambda} + 2(2n-1)} \right\}^2$$

If the value of  $L/\lambda$  is large relative to the number of half-period zones, then the expression for  $R(N)$  reduces to

$$R(N) \cong 4 N^2$$

Substituting the value for  $R(N)$  into the expression for gain, the new relation is

$$G = \frac{2(2L + N\lambda)}{N\lambda} 4 \left[ \sum_{n=1}^N \frac{\frac{8L}{\lambda} + (2n-1)}{\frac{8L}{\lambda} + 2(2n-1)} \right]^2$$

If the approximation is used, then the gain becomes

$$G = \frac{2(2L + N\lambda)}{N\lambda} 4 N^2$$

The design can now be approached in the following manner with reference to Fig. 5. Given: 1. frequency,  $f$ , 2. wavelength,  $\lambda$ , 3. diameter of plate,  $D$ , 4. focal length,  $L$ , and 5. relative permittivity of the material,  $\epsilon_r$ . Find: 1. half-power beamwidth  $BW = 2\theta = 2 \sin^{-1} 0.515 \lambda/D$ , 2. zone radii,  $r_n = \sqrt{N L \lambda + N^2 \lambda^2}/4$ , 3. number of zones, " $N$ ", 4. depth of groove (for folded zone plate only)  $d = \lambda/2(\sqrt{\epsilon_r}-1)$ , 5. gain of zone plate -- use the appropriate

expressions for gain with relation to the type of zone plate to be designed as previously discussed, and 6. bandwidth  $B \cong 2f/N$ .

The 10.5 inch Diameter Plate. -- The construction was that of a folded zone plate made of plexiglass with a maximum diameter of 10.5 inches. The design frequency was 19.3 Gc and the focal length 8 inches. Using the relations previously developed, the lens was constructed with the following parameters.

Beamwidth at the half-power points:  $3^{\circ} 36'$

Radii of the grooves:  $r_1 = 2.235$  in.,  $r_2 = 3.190$  in.,  $r_3 = 3.942$  in.,  $r_4 = 4.593$  in.,  $r_5 = 5.185$  in. From these,  $D_{\min.}$  is 10.37 in. Therefore:  $D$  is 10.50 in.

The number of zones: 5:  
Depth of groove: 0.455 in.:  
Gain of plate: 30.95 db.:  
Approximate bandwidth: 7.72 Gc.  
The plate was made from  $3/4$  in. stock of plexiglass with  $\epsilon_r$  of 2.8.

The radiation pattern of the 10.5 inch diameter zone plate is plotted in Fig. 6. The pattern remains the same irrespective of which way the grooves are facing. This was proved simply by taking the radiation pattern of the plate with the grooves facing one way, then reversing the plate and taking the radiation pattern again. The experimental value of the half-power beamwidth checks with the calculated value of  $3^{\circ} 36'$ . The same type of polarization measurement was made on the zone plate as was made on the dielectric lens. It was confirmed that the zone plate can accept any type of polarization of the wave.

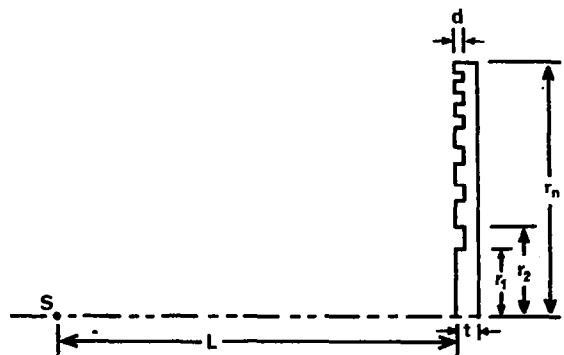


Figure 5 Fresnel Half-Period Zone Plate

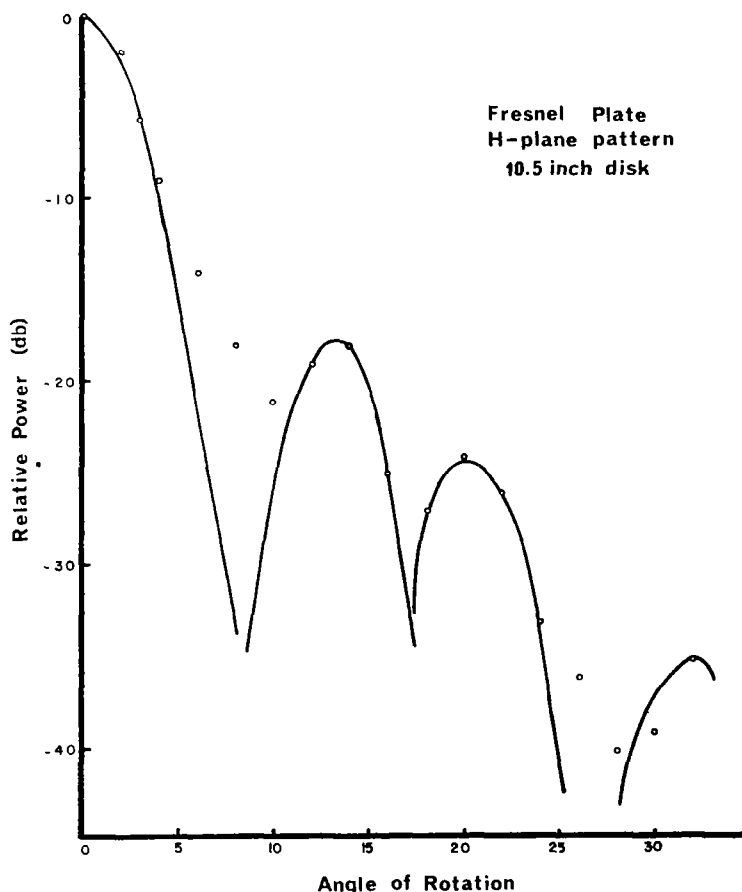


Figure 6 Fresnel Plate Radiation Pattern for 10.5 inch disk

The 22 Inch Diameter Plates. -- Two of these plates were constructed, one was made out of concentric aluminum strips that alternated with clear rings. These strips were glued onto a plexiglass sheet a 1/4 inch thick. The other plate was of the folded zone design also made out of plexiglass. Both zone plates have a focal length of 12 inches and operate at a frequency of 19.3 Gc. These two plates are the same as far as the radii, half-power beamwidth, and number of zones are concerned. They differ in their gains, and the folded plate has a groove depth associated with it. The plates were constructed with the following parameters:

Beamwidth at the half-power points:  $1.8^\circ$ . Radii of the zones:

$r_1 = 2.73$ in.	$r_5 = 6.26$ in.	$r_9 = 8.60$ in.
$r_2 = 3.89$ in.	$r_6 = 6.90$ in.	$r_{10} = 9.12$ in.
$r_3 = 4.79$ in.	$r_7 = 7.50$ in.	$r_{11} = 9.62$ in.
$r_4 = 5.57$ in.	$r_8 = 8.06$ in.	$r_{12} = 10.10$ in.

From these,  $D_{\min.}$  is 20.20 in.  
therefore:  $D$  is 22 in.

The number of zones: 12:  
Gain of the concentric ring plate: 26.22 db.: Gain of the folded zone plate: 36.32 db.:  
Depth of groove of the folded zone plate: 0.455 in.: Approximate bandwidth: 3.22 Gc.

The folded zone plate was made from 3/4 in. stock of plexiglass with  $\epsilon_r$  of 2.8. Two radiation patterns were obtained with the folded zone plate at distances of 60 feet and 10 feet, respectively (Figures 7 and 8). If the wave front were approximately plane, then the patterns should look the same neglecting of course any phase error that might be introduced. Upon observing the two patterns in Fig. 7 and 8, they are similar, although the pattern corresponding to the 10 foot distance seems to be a little more spread out. This is caused by some phase error in bringing the plates closer together. Since the beamwidths obtained from the radiation patterns agree with the calculated value of  $1.8^\circ$ , and the main lobes as well as the first side lobes

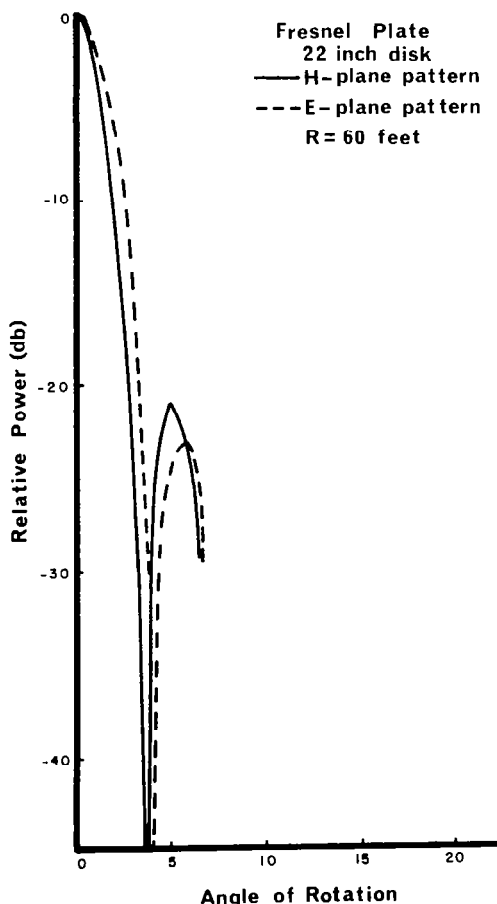


Figure 7 Fresnel Plate Radiation Pattern for 22 inch disk,  $R = 60$  feet

are similar, there must exist a fairly uniform plane wave between the plates. It follows that the radiation patterns of the two plates is the same since the diffracting action is the same in both cases.

The power distribution of both plates was measured and the results are plotted in Figure 9. The distributions are seen to be similar although they are not constant across the aperture. This effect is advantageous when the lens system is used as a probe since most of the power is concentrated near the axis of the system.

To compare the gain of each plate, one plate is used at the transmitter section, the other at the receiver section. Then gain measurements are made with and without the plate at the transmitter. With the concentric ring plate at the transmitter, the gain of the folded zone plate was measured at 22.5 db. With the folded zone plate at the transmitter, the gain measurement was 14.0 db. The calculated difference in gain was 10 db. as compared to the measured value of 8.5 db. The absolute measurement of gain by the reflection method was not made because the size of the reflecting surface as well as the distances involved would be too great.

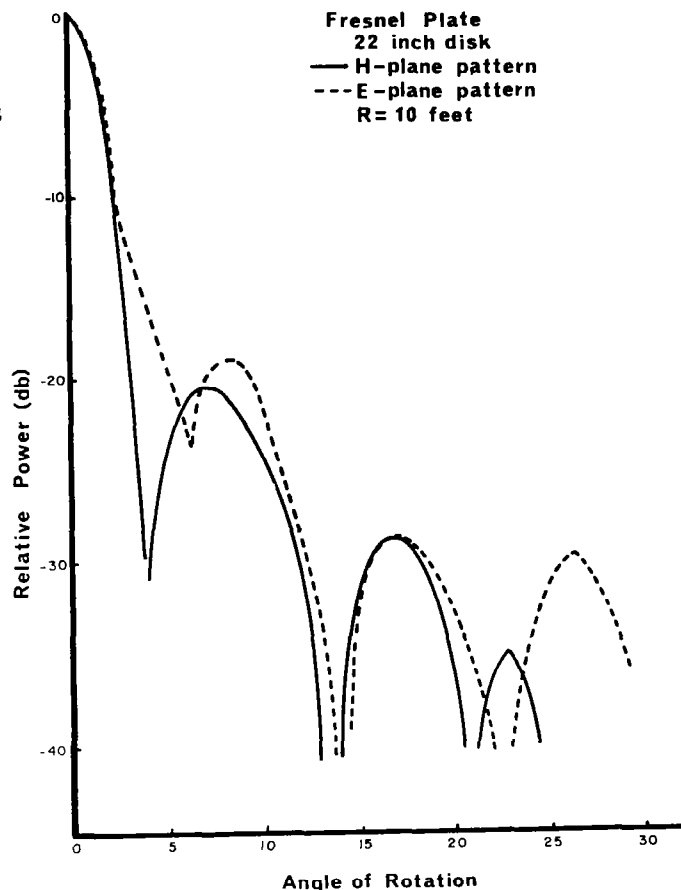


Figure 8 Fresnel Plate Radiation Pattern for 22 inch disk, R = 10 feet.

## CONCLUSIONS

From the results on the various radiators of the lens system, the best combination is that of the conical horn illuminating the large 22 inch Fresnel folded half-period zone plate. The results show that this lens system will perform acceptably at K-band frequencies. If the dimensions of the plasma remain the same, and the frequency is raised from K-band to the 4 mm. region, then the scaled version of this lens system should work very well.

The zone plate is preferred over the dielectric lens for the following reasons. In the first place, if a dielectric lens with a 22 inch diameter was to be constructed, the thickness would become excessive. This drawback could be overcome by making a stepped dielectric lens but this would be difficult to machine. Second, the Fresnel plate offers better control of beamwidth and gain. Third, it was desired that most of the microwave energy be concentrated around the axis through the center of the lens. This was accomplished better by the zone plate.

The zone plate lens system could easily be scaled down to operate at 4 mm. The diameter of the plate at this wavelength keeping the same bandwidth would be 5.14 inches. The focal length would also be reduced if the same number of zones are to be kept. Since the size of the plasma cell is roughly 3 inches in diameter, such a probe could actually be used to probe the structure of the plasma. Our present K-band lens system cannot. This does not mean that the K-band system will not work; it means that the entire plasma interacts with the beam. In the K-band case, we are making an average measurement on the plasma while, if the lens system is used in the 4 mm. region, it will truly be a probe.

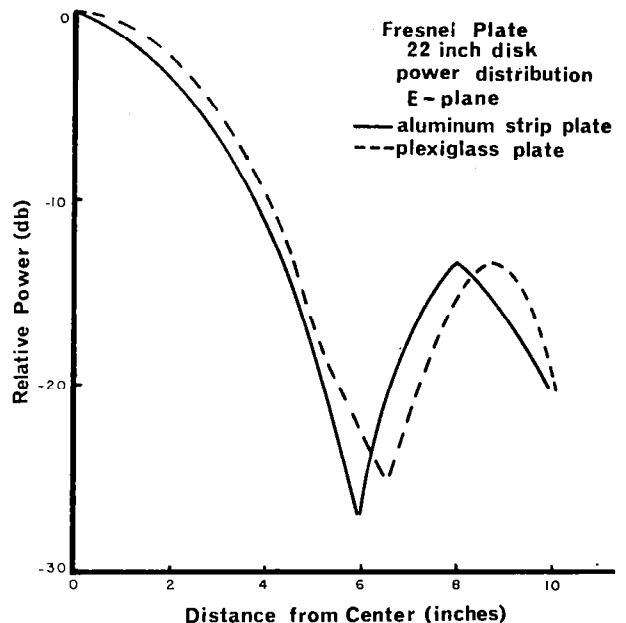


Figure 9 Fresnel Plate Power Distribution for 22 inch disk

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